On obtaining OWA operator weights: a short survey of recent developments

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***Abstract*— The determination of ordered weighted averaging (OWA) operator weights is a very important issue of applying the OWA operator for decision making. One of the first approaches, suggested by O'Hagan, determines a special class of OWA operators having maximal entropy of the OWA weights for a given level of *orness*; algorithmically it is based on the solution of a constrained optimization problem. In 2001, using the method of Lagrange multipliers, Fullér and Majlender solved this constrained optimization problem analytically and determined the optimal weighting vector. In 2003 Fullér and Majlender also suggested a minimum variance method to obtain the minimal variability OWA operator weights. In this paper we give a short survey of some later works that extend and develop these models.**

I. Obtaining OWA operator weights

An OWA operator of dimension n is a mapping $F\colon \mathbb{R}^n \to \mathbb{R}$ that has an associated weighting vector $W = (w_1, \dots, w_n)^T$ of having the properties

$$w_1 + \cdots + w_n = 1,\ 0 \leq w_i \leq 1,\ i = 1, \dots, n,$$

and such that

$$F(a_1, \dots, a_n) = \sum_{i=1}^{n} w_i b_i,$$

where b_j is the jth largest element of the collection of the aggregated objects $\{a_1, \dots, a_n\}$.

In [4], Yager introduced two characterizing measures associated with the weighting vector W of an OWA operator. The first one, the measure of *orness* of the aggregation, is defined as

$$\mathrm{orness}(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i.$$

and it characterizes the degree to which the aggregation is like an *or* operation. It is clear that $\mathrm{orness}(W) \in [0,1]$ holds for any weighting vector.

The second one, the measure of *dispersion* of the aggregation, is defined as

$$\mathrm{disp}(W) = -\sum_{i=1}^{n} w_i \ln w_i$$

and it measures the degree to which W takes into account all information in the aggregation.

It is clear that the actual type of aggregation performed by an OWA operator depends upon the form of the weighting vector. A number of approaches have been suggested for obtaining the associated weights, i.e., quantifier guided aggregation [4], [5], exponential smoothing [7] and learning [6].

O'Hagan [1] determined OWA operator weights and suggested a maximum entropy method, which formulates the OWA operator weight problem as a constrained nonlinear optimization model with a predefined degree of orness as its constraint and the entropy as its objective function. This approach is based on the solution of he following mathematical programming problem:

$$\text{maximize}\ -\sum_{i=1}^{n} w_i \ln w_i$$

$$\text{s.t.}\ \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i = \alpha,\ 0 \leq \alpha \leq 1$$

$$\sum_{i=1}^{n} w_i = 1,\ 0 \leq w_i \leq 1,\ i = 1, \dots, n.$$

In 2001, using the method of Lagrange multipliers, Fullér and Majlender [2] solved this constrained optimization problem analytically and determined the optimal weighting vector. By their method, the associated weighting vector is easily obtained by

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1$$

$$w_j = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}}$$

and

$$w_n = \frac{((n-1)\alpha - n) w_1 + 1}{(n-1)\alpha + 1 - n w_1}$$

then

$$w_1[(n-1)\alpha + 1 - n w_1]^n = ((n-1)\alpha)^{n-1} [((n-1)\alpha - n) w_1 + 1]$$

In 2003 Fullér and Majlender [3] also suggested a minimum variance method to obtain the minimal variability OWA operator weights. A set of OWA operator weights with minimal variability could then be generated. Their approach requires the solution of the following mathematical programming problem:

$$\text{minimize } D^2(W) = \frac{1}{n} \cdot \sum_{i=1}^{n} \left(w_i - \frac{1}{n} \right)^2$$

$$\text{s.t. orness}(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \ 0 \leq \alpha \leq 1,$$
$$w_1 + \cdots + w_n = 1, \, 0 \leq w_i \leq 1, \, i = 1, \ldots, n.$$

II. Extensions

In 2004 Liu and Chen [37] showed the equivalence of geometric OWA operator and maximum entropy OWA operator weights.

In 2005 Wang and Parkan [35] presented a minimax disparity approach, which minimizes the maximum disparity between two adjacent weights under a given level of orness. Their approach was formulated as:

$$\text{minimize } \max_{i=1,2,\ldots,n-1} | \, w_i - w_{i+1} \, |$$

$$\text{s.t. orness}(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \ 0 \leq \alpha \leq 1,$$
$$w_1 + \cdots + w_n = 1, \, 0 \leq w_i \leq 1, \, i = 1, \ldots, n.$$

In 2005 Majlender [6] developed a maximal Rényi entropy method for generating a parametric class of OWA operators and the maximal Rényi entropy OWA weights. His approach was formulated as:

$$\text{maximize } H_\beta(w) = \frac{1}{1-\beta} \log_2 \sum_{i=1}^{n} w_i^\beta$$

$$\text{s.t. orness}(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \ 0 \leq \alpha \leq 1,$$
$$w_1 + \cdots + w_n = 1, \, 0 \leq w_i \leq 1, \, i = 1, \ldots, n.$$

where $\beta \in \mathbb{R}$ and $\mathrm{H}_1(w) = -\sum_{i=1}^{n} w_i \log_2 w_i$.

In 2007 Liu [18] proved that the solutions of the minimum variance OWA operator problem under given orness level and the minimax disparity problem for OWA operator are equivalent, both of them have the same form of maximum spread equidifferent OWA operator. He also introduced the concept of maximum spread equidifferent OWA operator and proved its equivalence to the minimum variance OWA operator.

In 2007 Wang et al [12] introduces two models determining as equally important OWA operator weights as possible for a given orness degree. Their models can be written as

$$\text{minimize } J_1 = \sum_{i=1}^{n-1} (w_i - w_{i+1})^2$$

$$\text{s.t. orness}(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \ 0 \leq \alpha \leq 1,$$
$$w_1 + \cdots + w_n = 1, \, 0 \leq w_i \leq 1, \, i = 1, \ldots, n.$$

and

$$\text{minimize } J_2 = \sum_{i=1}^{n-1} \left(\frac{w_i}{w_{i+1}} - \frac{w_{i+1}}{w_i} \right)^2$$

$$\text{s.t. orness}(w) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \ 0 \leq \alpha \leq 1,$$
$$w_1 + \cdots + w_n = 1, \, 0 \leq w_i \leq 1, \, i = 1, \ldots, n.$$

In 2007 Llamazares [11] proposed determining OWA operator weights regarding the class of majority rule that one should want to obtain when individuals do not grade their preferences between the alternatives. In 2005 Cheng et al [44] evaluated airline service quality uisng OWA operators.

The OWA weighting vector and the fuzzy quantifiers are strongly related. An intuitive way for shaping a monotonic quantifier, is by means of the threshold that makes a separation between the regions of what is satisfactory and what is not. Therefore, the characteristics of a threshold can be directly related to the OWA weighting vector and to its metrics: the attitudinal character and the entropy. Usually these two metrics are supposed to be independent, although some limitations in their value come when they are considered jointly. In 2005 Troiano and Yager [45] argued that these two metrics are strongly related by the definition of quantifier threshold, and theyshowed how they can be used jointly to verify and validate a quantifier and its threshold.

In 2006 Xu [42] investigated the dependent OWA operators, and developed a new argument-dependent approach to determining the OWA weights, which can relieve the influence of unfair arguments on the aggregated results.

In 2006 Zadrozny and Kacprzyk [46] discussed the use of the Yagers OWA operators within a flexible querying interface. Their key issue is the adaptation of an OWA operator to the specifics of a users query. They considered some well-known approaches to the manipulation of the weights vector and proposed a new one that is simple and efficient. They discussed the tuning (selection of weights) of the OWA operators, and proposed an algorithm that is effective and efficient in the context of their FQUERY for Access package [9], [10].

In 2006 [29] Chang et al proposed a dynamic fuzzy OWA model to deal with problems of group multiple criteria decision making. Their proposed model can help users to solve MCDM problems under the situation of fuzzy or incomplete information.

In 2006 Wang et al [27] developed the query system of practical hemodialysis database for a regional hospital in Taiwan, which can help the doctors to make more accurate decision in hemodialysis. They built the fuzzy membership function of hemodialysis indices based on experts interviews. They proposed a fuzzy OWA query method, and let the decision makers (doctors) just need to change the weights of attributes dynamical, then the proposed method can revise the weight of each attributes based on aggregation situation and the system will provide synthetic suggestions to the decision makers.

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